[15]; Eckert and Drake [16], Tables B3, B4); Table 1 summarizes the results. In nondimensional form, this relation becomes $f = \tau^{\omega}$; substitution of the latter into (9) yields:

$$F = \frac{2(1-\tau_b^{\omega+1})}{(\omega+1)(1-\tau_b)}; \quad \omega \neq -1$$
$$F = \frac{2}{1-\tau_b}\ln(\tau_b^{-1}); \quad \omega = -1. \tag{10}$$

Equation (10) therefore gives the dependence of both total and local heat transfer on ω, for an object of any shape having an isothermal surface. Figure 1 shows this dependence for various values of τ_b.

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A CONSTANT HEAT FLUX MODEL OF THE EVAPORATING INTERLINE REGION

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NOMENCLATURE

$\bar{A}$,	dispersion constant [J];
h,	heat-transfer coefficient [$\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$];
h_{fg},	latent heat of vaporization [$\text{J}\cdot\text{kg}^{-1}$];
k,	thermal conductivity [$\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$];
M,	molecular weight [$\text{kg}\cdot\text{mol}^{-1}$];
$\dot{m}$,	mass flux [$\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$];
P,	pressure [$\text{N}\cdot\text{m}^{-2}$];
Q,	heat transferred [$\text{W}\cdot\text{m}^{-1}$];
q,	heat flux [$\text{W}\cdot\text{m}^{-2}$];
R,	universal gas constant [$\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$];
S,	solid thickness [m^{-1}];
T,	temperature [K];
U,	overall heat-transfer coefficient, [$\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$];
V,	molar volume [$\text{m}^3\cdot\text{mol}^{-1}$].

Greek symbols

γ,	dimensionless heat-transfer coefficient;
δ,	film thickness [m];
η,	dimensionless film thickness;
ν,	kinematic viscosity [$\text{m}^2\cdot\text{s}^{-1}$];
ξ,	dimensionless film length coordinate;
σ,	evaporation coefficient [dimensionless].

Subscripts and superscripts

l,	liquid phase;
lv,	liquid–vapor interface;
s,	solid;
t,	constant temperature solution (1);
0.	evaluated at interline;
v,	vapor phase;
$-$,	averaged;
id,	ideal;
$*$.	differentiation with respect to ξ.

1. INTRODUCTION

IN A previous paper [1], a procedure to determine the theoretical heat-transfer coefficient for the interline region (junction of vapor, evaporating thin film and non-evaporating thin film) of a wetting film was developed. Since the analysis was based on the assumption of a constant liquid–vapor interfacial temperature, the effect of the thermal conductivity of the solid on the process was not included. In this communication an approximate procedure to determine the heat sink capability of the interline region is developed in which the effect of the solid resistance to heat transfer is included. The formulation uses a simple one-dimensional constant heat flux model. Although the general approach used in [1] is continued, the solution herein is directed towards a limiting case in which the solid conductance is controlling. Since the value of the ideal liquid–vapor interfacial heat-transfer coefficient is very large, solid conductance is found to dominate the overall heat-transfer coefficient in the interline region after a small increase in the film thickness. The results are useful for predicting and comparing the heat sink capability of various liquid–solid systems.

2. DEVELOPMENT OF THE EQUATIONS

The system to be studied consists of a very thin, evaporating film of pure liquid on a horizontal solid substrate of thickness S (see Fig. 1). The shape of the thin film is steady since the evaporating liquid is replenished by fluid flow towards the interline from a source outside of the analyzed region. The temperature of the solid surface at $y = -S$ is a constant, T_s, which is greater than the vapor temperature, T_v.

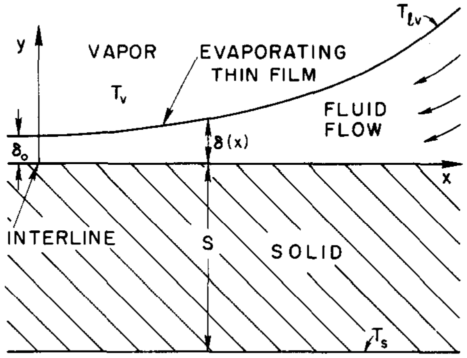

FIG. 1. Interline junction of vapor, evaporating thin film and non-evaporating thin film (not to scale).

If the film is sufficiently thin, it is kept from evaporating by London–van der Waals dispersion forces between the solid and liquid. If a section of the film is slightly tapered, portions of the film may be sufficiently thick to allow evaporation. The junction between the non-evaporating and the evaporating thin film is the interline. Fluid flows in the negative "X" direction in the evaporating thin film as a result of a pressure gradient in the film resulting from the film thickness gradient.

Using the procedure developed in [1] the evaporative heat flux is

$$q = h_{fg}\dot{m}_e = \frac{h_{fg}\bar{A}}{\nu}\left(\frac{\delta'}{\delta}\right)'.$$

For the present case q is assumed to be a constant $\bar{q}$ and equal to the average macroscopic heat flux through the system:

$$U(T_s - T_v) = \frac{h_{fg}\bar{A}}{\nu}\left(\frac{\delta'}{\delta}\right)' \tag{2}$$

where, the average overall heat-transfer coefficient U, is defined as

$$U = \left[\frac{S}{k_s} + \frac{\bar{\delta}}{k_l} + \frac{1}{\bar{h}_{lv}}\right]^{-1} \approx \frac{k_s}{S} \tag{3}$$

in which, $\bar{h}_{lv}$ is the average liquid–vapor interfacial heat-transfer coefficient and $\bar{\delta}$ is the average thickness of the thin film. Equation (2) is made dimensionless by using the previously defined variables (1):

$$\eta = \delta/\delta_0; \quad \xi^2 = GX^2; \quad \eta^* = \mathrm{d}\eta/\mathrm{d}\xi.$$

$$\left(\frac{\eta^*}{\eta}\right)^* = \frac{U\nu}{h_{fg}\bar{A}G}(T_s - T_v) \tag{4}$$

where

$$G = \frac{\nu}{\bar{A}}\left(\frac{2\sigma}{2-\sigma}\right)\left(\frac{M}{2\Pi R\bar{T}}\right)^{1/2}\left(\frac{P_v M h_{fg}}{R}\right)\left(\frac{T_{lv} - T_v}{T_{lv}T_v}\right)_0 \tag{5}$$

and the interline thickness δ_0, is given by

$$\delta_0 = \left(\frac{\bar{A}V_l T_v}{Mh_{fg}(T_{lv} - T_v)_0}\right)^{1/3} \tag{6}$$

Since

$$\frac{h_{fg}\bar{A}G}{\nu} = h_{lv}^{id}(T_{lv} - T_v)_0 = q^{id} \tag{7}$$

this group gives the ideal heat flux based on the kinetic theory of vapor flow that would occur at a liquid–vapor interface due to the temperature differences, $(T_{lv} - T_v)_0$. Using equations (4)–(7) gives

$$\left(\frac{\eta^*}{\eta}\right)^* = \frac{\bar{q}}{q^{id}} = \bar{\gamma}. \tag{8}$$

Solving equation (8) for constant $\bar{\gamma}$ with the boundary conditions:

$$\eta(0) = 1; \quad \frac{\mathrm{d}\eta}{\mathrm{d}\xi}\Big|_0 = \eta_0^* \tag{9}$$

we obtain

$$\ln(\eta) = \frac{\bar{\gamma}\xi^2}{2} + \eta_0^*\xi. \tag{10}$$

For the present case, the dimensionless contact angle is taken to be zero, $\eta_0^* = 0$, and the extent of the region of evaporation ξ, is

$$\xi = \left[\frac{2\ln\eta}{\bar{\gamma}}\right]^{0.5} \tag{11}$$

or

$$x = \left[\frac{2\ln\eta}{G\bar{\gamma}}\right]^{0.5} = \left[\frac{2h_{fg}\bar{A}\ln\eta}{\bar{q}\nu}\right]^{0.5}. \tag{12}$$

The size of the interline heat sink Q, with a film thickness $\eta > 1$, is

$$Q = \bar{q}x = q^{id}\left[\frac{2\gamma\ln\eta}{G}\right]^{0.5}. \tag{13}$$

Using the previous definitions, equation (13) becomes

$$Q = [2Uh_{fg}\bar{A}(T_s - T_v)\nu^{-1}\ln\eta]^{0.5} \tag{14}$$

the solution for the limiting case of constant interfacial temperature (1) is

$$Q = h_{fg}\bar{A}G^{0.5}\nu^{-1}[1 - 0.5(\eta^{-1} + \eta^{-2})]\xi_t(\eta). \tag{15}$$

To compare the heat sink capability Q, of various systems using equation (14) with equal dimensionless thickness η, the parameter $[2Uh_{fg}\bar{A}(T_s - T_v)/\nu]$, is important. This includes the usual dimensions of the system and the properties of the liquid and solid. In addition, it includes the dispersion constant $\bar{A}$, which accounts for the London–van der Waals dispersion force between the solid and liquid. Therefore, the relative effect of the classical overall heat-transfer coefficient U, the "surface transfer coefficient" $\bar{A}/\nu$ and the "interline heat flow number" $h_{fg}\bar{A}\nu^{-1}$, can be evaluated. This also defines an effective heat-transfer coefficient $2U\bar{A}h_{fg}/\nu(T_s - T_v)]^{0.5}$, based on the film thickness range $\eta > 1$.

It is not possible to increase the heat flux without bound as suggested by the equations because the system would become unstable. This can be seen by rewriting equation (7) as

$$U(T_s - T_v) = \bar{\gamma}(T_{lv} - T_v)_0 h_{lv}^{id}. \tag{16}$$

For the case of constant liquid–vapor interfacial temperature (negligible solid resistance) the heat flux ratio $\bar{\gamma}$, has a limiting value of

$$\bar{\gamma} = 1 - 0.5(\eta^{-1} + \eta^{-2}). \tag{17}$$

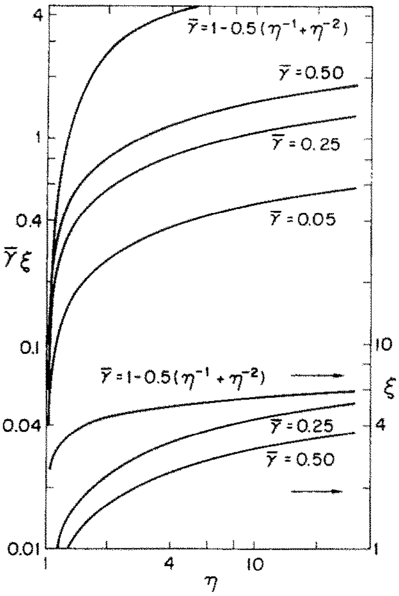

FIG. 2. Dimensionless heat sink profiles $\bar\gamma\xi$ and dimensionless film profiles ξ.

For the constant heat flux case $\bar\gamma$, would be less than this value. The limit for the value of $(T_{lv} - T_v)_0$ depends on the ability of the evaporating film profile to change and provide the necessary pressure gradient for flow which is finite.

In Fig. 2 the dimensionless heat sink capability $\bar\gamma\xi$ is presented as a function of the dimensionless thickness η. The value of the heat sink for the constant liquid–vapor interfacial temperature case starts below the constant heat flux case because of effect of the dispersion force on vapor pressure. However, it surpasses the constant heat flux case at a very small dimensionless thickness. The thin film profiles for three values of $\bar\gamma$ are also presented in Fig. 2. Combining the results of the two limiting cases for a closer approximation of the real physical situation, we can say, qualitatively, that the initial portion of the evaporating thin film follows the constant temperature model until thermal resistance limits the heat flux to a constant value. This limit occurs at a very small increase in film thickness.

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NOTE ON DIFFUSION IN A TURBULENT BOUNDARY LAYER

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NOMENCLATURE

C_f,	skin friction coefficient;
c_p,	specific heat at constant pressure;
k,	effective turbulent Prandtl number near wall;
l,	characteristic length of source;
q,	strength of heat source per unit length;
u,	velocity in x direction;
x,	distance from the origin of the turbulent boundary layer;
x',	distance from the source;
y,	distance normal to the wall;
$y_{1/2}$,	distance where $\theta = \theta_w/2$;
$\bar y$,	mean diffusion distance normal to the wall.

Greek symbols

δ^*,	boundary layer displacement thickness;
ε,	eddy diffusivity;
θ,	temperature difference referenced to the free-stream temperature;
ν,	kinematic viscosity;
ρ,	density;
τ_w,	wall shear stress.

Subscripts

0,	at the source;
w,	at the wall;
∞,	in the free stream.

THIS note concerns one aspect of turbulent flow which has received repeated attention—diffusion in a turbulent boundary layer downstream of a line source of heat (or mass) positioned on an adiabatic (or impervious) surface. In particular, two existing analyses of this problem which use substantially different eddy diffusivity distributions are compared to published experimental results. The purpose of the note is to show that these theories successfully describe the mean temperature (or concentration) distribution downstream of the source providing a near and far field approach is taken.

Consider a line source of heat placed on an adiabatic wall and at right angles to the flow. If the characteristic source dimension l, is less than the linear sublayer thickness and the buoyancy force near the source is small compared to the inertia force in the sublayer, i.e. the source Grashof number is much less than $l^2\tau_w/\rho\nu^2$, where $(\tau_w/\rho)^{1/2}$ and ν are the friction velocity and kinematic viscosity, respectively, the flow downstream of the source can be considered to develop as an undisturbed turbulent boundary layer.

Near the source, diffusion is controlled by the turbulence structure near the wall and providing the source is located

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